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OPTIMAL DESIGN OF LOCALLY ORTHOTROPIC ELASTIC FLAT BODIES WITH WEAK BINDING

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Optimal design of elastic bodies made of fiberglass reinforced plastics or metals reinforced by fibers with a high elasticity modulus. It is assumed that the composite components are in a plane strain or a plane stress state and that the reinforcement is directed along two mutually orthogonal families of plane curves. The equations of these curves and the reinforcement level at each point of the bulk body are determined for a given boundary profile and specified conditions existing on this boundary.
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Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

* ye initially, after vowels, and after ъ, ь; e elsewhere. When written as ё in Russian, transliterate as yё or ё. The use of diacritical marks is preferred, but such marks may be omitted when expediency dictates.

FOLLOWING ARE THE CORRESPONDING RUSSIAN AND ENGLISH DESIGNATIONS OF THE TRIGONOMETRIC FUNCTIONS

Russian	English
sin	sin
cos	cos
tg	tan
ctg	cot
sec	sec
cosec	csc
sh	sinh
ch	cosh
th	tanh
cth	coth
sch	sech
csch	csch
arc sin	$\sin^{-1}$
arc cos	$\cos^{-1}$
arc tg	$\tan^{-1}$
arc ctg	$\cot^{-1}$
arc sec	$\sec^{-1}$
arc cosec	$\csc^{-1}$
arc sh	$\sinh^{-1}$
arc ch	$\cosh^{-1}$
arc th	$\tanh^{-1}$
arc cth	$\coth^{-1}$
arc sch	sech^{-1}
arc csch	csch^{-1}
rot	curl
lg	log

OPTIMAL DESIGN OF LOCALLY ORTHOTROPIC ELASTIC FLAT BODIES WITH WEAK BINDING

G. I. Bryzgalin

(Volgograd)

Solved are problems of optimal design for elastic bodies made from a material such as fiberglass reinforced plastics or metal reinforced by fibers with a high elasticity modulus [1, 2]. It is assumed that the composite components are in a plane strain or a plane stress state and the reinforcement is directed along two mutually orthogonal families of plane curves. The equations for these curves, the reinforcement strength at each point, and the body thickness are determined during the designing for a given boundary profile and conditions on it.

Let us present a rather small body element cut by planes normal to the directions of reinforcement. On the two opposite edges orthogonal to the first direction of reinforcement we will apply expanding or compressing normal stresses σ_1. We will assume that the material is deformed as a monolith, i.e., deformations ε_1 along the stress, on the average, are equal at all points. Let σ_1' be the stress in the reinforcement fibers of the first direction; s_1 – cross sectional area of these fibers per unit area of material; and E' – elasticity modulus of fibers.

If the edge of the examined element has a unit area, then the remaining part of material, composed of binder layers and fibers oriented in a transverse direction, will occupy on it area $1 - s_1$. The average elasticity modulus of this remaining part is designated in terms of E'', and the average stress – σ_1''. Using the well-known formula of mechanical mixing we obtain

$$\sigma_1 = \sigma_1' s_1 + \sigma_1''(1 - s_1), \quad \sigma_1' = E'\varepsilon_1, \quad \sigma_1'' = E''\varepsilon_1 \tag{1}$$

Considering that s_1 does not equal zero and E'' is considerably less than E', we can write the approximate equality

$$\sigma_1 = \sigma_1' s_1 = E' s_1 \varepsilon_1 \tag{2}$$

Calculations based on more specific formulas [3] indicate that if the modulus of elasticity of binder E^c is ten times smaller than the reinforcement modulus, then the replacement of equality (1) with equality (2), at worst, yields an error (contributing to the safety factor) on the order of 30% of the maximally possible stress in the material.

The following experimental factors also speak in favor of this substitution:

a) the elasticity modulus of the binder of fiberglass reinforced plastics is 10-20 times lower than the elasticity modulus of reinforcement;

b) usually, binders are subject to considerable creep, and coefficient E'' in the preceding formulas, in essence, is not a constant value but an operator, such that with ε_1 = const $\lim E''\varepsilon_1 = 0$ when $t \to \infty$ (t – time). So that for parts working under conditions of prolonged action of slightly changing stresses, relationship (2) can prove to be more acceptable than (1);

c) the range of the elastic and strength curves of composite materials is tens of percent from sample to sample.

The elasticity modulus of such an element can be considered as identical during expansion and compression [4], and Poisson coefficients during deformation along the fibers as small (so small that they can be disregarded); therefore, we extend the relationship of type (2) to the general case of plane stressed state.

We will limit ourselves to the designs in which the main axes of stress and strain tensors coincide with the fiber directions of reinforcement at every composite point (rational designs [2]). Then in the curvilinear orthogonal system of coordinates $\alpha\beta$ along whose coordinate lines the reinforcing fibers are placed, the following law governing the connection between strain and stresses can be written as:

$$\sigma_1 = E s_1 \varepsilon_1, \qquad \sigma_2 = E s_2 \varepsilon_2 \tag{3}$$

Here σ_1, σ_2 – principle stresses, ε_1, ε_2 – principle strains, E – constant selected on the basis of experiments on the composite material.

The problem concerning the strength criterion of such hypothetical material, in other words, concerning the limits of applicability of relationships (3) is rather complex and it is doubtful whether it can have a unique answer suitable for description of all actual materials of a similar type.

Taking advantage of the fact that tensile strengths of composite materials during extension and compression are values of the same order, we will assume that the permissible state of an element is characterized by deformation values not exceeding a certain maximum value of

$$|\varepsilon_1| \leq \varepsilon, \qquad |\varepsilon_2| \leq \varepsilon \tag{4}$$

For reinforcement which remains elastic right up to destruction ($\sigma_i' = E'\varepsilon_i$, $i = 1, 2$), this means that the maximum value of internal stress in fibers determines the material strength.

Of course, during compression or during a complex stressed state, destruction occurs not necessarily due to a break in fibers; however, criterion (4) can be used safely if the materials tested have different s_1, s_2, if we construct families of strength curves at different stressed states in plane $\varepsilon_1 \varepsilon_2$, and select the square of the permissible states such that none of the strength curves should intersect it (Fig. 1).

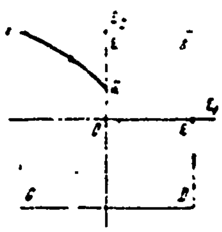

Fig. 1.

Since there are no fibers in a direction normal to the coordinate lines of system $\alpha\beta$, then within the accepted permissible limits we can assume that the third principal stress is equal to zero $\sigma_3 = 0$. The stress components with mixed indexes equal zero due to the fact that this is precisely how the rational designs are constructed; so that directions α = const and β = const were the principal tensions of stress and strain tensors $\sigma_{12} = \sigma_{13} = \sigma_{23} = 0$.

In view of this fact there will not be any fundamental difference between the plane stress and plane strain states. We will only note that with a plane strain state the design is for a composite layer with a unitary thickness, while with a plane stress state – the thickness can be arbitrary and even vary from point to point; thus, only its minimally possible value is selected during designing. The problems will become wholly identical, if in the latter case in place of stresses and levels of reinforcement we introduce values

[illegible] (5)

(h – composite thickness at a given point)

If we consider that the above examined volume element has height h and unitary transverse dimensions, then T_1, T_2 – forces acting on this element along directions α and β; S_1, S_2 – fiber volumes of corresponding direction in it. (For simplicity, T_1, T_2 will also be called stresses.)

The equations of equilibrium of such volume element ([5], Chapter 2) can be obtained from (8.2), if we assume that, there, $\sigma_{12} = 0$

$$\frac{\partial}{\partial\alpha}(H_2 T_1) - \frac{\partial H_2}{\partial\alpha} T_2 = 0, \quad \frac{\partial}{\partial\beta}(H_1 T_2) - \frac{\partial H_1}{\partial\beta} T_1 = 0 \tag{6}$$

(H_1, H_2 – Lamé parameters of system $\alpha\beta$)

Formula (8.2) in work [6] (Chapter 2), when $\varepsilon_{12} = 0$, leads to the following conditions of deformation consistency:

$$\begin{aligned} &-2\left\{\frac{\partial^2}{\partial\beta^2}[H_1^2(1+2\varepsilon_1)] - \frac{\partial^2}{\partial\alpha^2}[H_2^2(1+2\varepsilon_2)]\right\} H_1^2 H_2^2(1+2\varepsilon_1)(1+2\varepsilon_2) + \\ &+ \left\{\left(\frac{\partial}{\partial\beta}[H_1^2(1+2\varepsilon_1)]\right)^2 + \frac{\partial}{\partial\alpha}[H_1^2(1+2\varepsilon_1)]\frac{\partial}{\partial\alpha}[H_2^2(1+2\varepsilon_2)]\right\} H_2^2(1+2\varepsilon_2) + \\ &+ \left\{\left(\frac{\partial}{\partial\alpha}[H_2^2(1+2\varepsilon_2)]\right)^2 + \frac{\partial}{\partial\beta}[H_2^2(1+2\varepsilon_2)]\frac{\partial}{\partial\beta}[H_1^2(1+2\varepsilon_1)]\right\} H_1^2(1+2\varepsilon_1) = 0 \end{aligned} \tag{7}$$

Equations (6), (7) should include boundary conditions on body profile, conditions (4), and positive conditions of reinforcement levels $S_1 \geq 0$, $S_2 \geq 0$.

Let the problem be solved with regard to the theory of elasticity for an isotropic body whose middle plane occupies area Ω bound by contour L on plane $\alpha\beta$. Then for constructing a reinforced body design with the same boundary profile and same conditions on it, we can assume that the reinforcement fibers are directed along the lines of principal stresses of the isotropic body (theorem on the existence of rational design [2]). For the selected system of coordinates $\alpha\beta$ (whose coordinate lines coincide with the isostatic grid of the isotropic body) we have nine equations (3), (5)-(7) and ten functions

$$T_1,\ T_2,\ \sigma_1,\ \sigma_2,\ S_1,\ S_2,\ s_1,\ s_2,\ \varepsilon_1,\ \varepsilon_2 \tag{8}$$

The initial form of these functions and the body thickness are determined according to the indicated theorem, and beyond this it is possible to state a variation problem, i.e., require that functions (8) yield an extremum to a certain functional. In this work, the design withstanding given loads and having the smallest

volume of reinforcement among all rational designs we will consider as optimum. In other words, for the optimum design, functions (8) satisfy all the conditions written earlier and yield a minimum for the following functional:

$$\int_{\Omega} (S_1 + S_2) d\Omega = \min \tag{9}$$

In a specific case, when both principal stresses in an isotropic body have the same sign, the design is constructed on the basis of the corresponding theorem of existence, in this case

$$\varepsilon_1 = \varepsilon_2 = \pm\varepsilon = \mathrm{const} \tag{10}$$

Another specific case corresponding to the following conditions:

$$\varepsilon_1 = -\varepsilon_2 = \pm\varepsilon = \mathrm{const} \tag{11}$$

is indicated in work [7]. The possibility of constructing a design which would satisfy equalities (11) is restricted by the condition of deformation consistency. Actually, assuming that ε_1 = const, ε_2 = const in (6) and using the condition of equality to zero of the tensor of curvature of space $\alpha\beta$ in the initial state, we find

$$H_1 \frac{\partial^2 H_2}{\partial \alpha^2} - \frac{\partial H_1}{\partial \alpha}\frac{\partial H_2}{\partial \alpha} = 0, \qquad H_2 \frac{\partial^2 H_1}{\partial \beta^2} - \frac{\partial H_1}{\partial \beta}\frac{\partial H_2}{\partial \beta} = 0 \tag{12}$$

In this case the second equality proves to be a symmetrical substitution of indexes and coordinates; furthermore, from it we can easily obtain

$$\partial H_1 / \partial \beta = f(\alpha) H_2 \tag{13}$$

where $f(\alpha)$ – arbitrary function. Using the known expressions for the curvatures of families of plane curves

$$\varkappa_1 = -\frac{1}{H_1 H_2}\frac{\partial H_1}{\partial \beta}, \qquad \varkappa_2 = \frac{1}{H_1 H_2}\frac{\partial H_2}{\partial \alpha}$$

we rewrite (13) thus

$$-\varkappa_1 H_1 = f(\alpha) \tag{14}$$

Condition (14) was shown in [7]; geometrically it means that the increase in the inclination angle of a tangent to line α per single increase of coordinate α does not depend on β.

Generally, we have to solve the variation problem; moreover, we should assume that functions T_2, S_2 can have a break along line α = const, and functions T_1, S_1 – along line β = const. After we determine functions S_1, S_2, thickness h can be found in the following manner. Let

$$\max_{\Omega} (S_1(\alpha, \beta) + S_2(\alpha, \beta)) = N$$

At point (α_0, β_0), where this maximum is achieved, we will assume that $s_1 + s_2 = s_*$ (s_* – greatest possible reinforcement density, $s_* \leq 1$). Then

$$h = N/s_*, \quad s_1 = s_* S_1/N, \quad s_2 = s_* S_2/N \tag{15}$$

There are other possibilities for selecting the thickness and level of reinforcement, since addition or removal of excess binder does not (within the framework of this system) affect the carrying ability of the design.

Further, we are presenting examples illustrating various possibilities of spatial optimum designs.

A. *Half-plane loaded on the edge section by a normal uniform force of intensity P.* First of all let us investigate a stressed state in the corresponding isotropic body. From formulas (4, 4', 5) of work [8], (page 352) we can obtain the following expression for the tensor components of stresses in an elastic isotropic half-plane loaded in the same way (there is an error in the corresponding equalities [8])

$$\begin{aligned} T_x &= \sigma_x h = \pi^{-1}P[-(\theta_1 - \theta_2) + 2xy\cos(\theta_1 + \theta_2)/r_1r_2] \\ T_y &= \sigma_y h = \pi^{-1}P[-(\theta_1 - \theta_2) - 2xy\cos(\theta_1 + \theta_2)/r_1r_2] \\ T_{xy} &= \sigma_{xy} h = -2\pi^{-1}Pxy\sin(\theta_1 + \theta_2)/r_1r_2 \end{aligned} \tag{16}$$

(notations are in Fig. 2). in this case it turns out that the

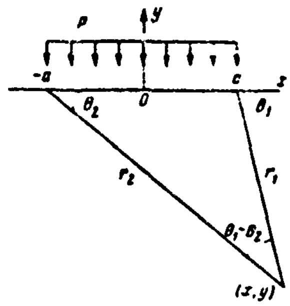

Figure 2.

principal direction α forms angle $\gamma = 1/2(\theta_1 + \theta_2)$ with axis x, and the principal stresses are

$$T_1 = \pi^{-1}P[-\theta + 2ay / r_1r_2], \quad T_2 = \pi^{-1}P[-\theta - 2ay / r_1r_2] \tag{17}$$

Here

$$\theta = \theta_1 - \theta_2, \quad \theta_1 = \text{arc tg}[-y / (x - a)], \quad \theta_2 = \text{arc tg}[-y / (x + a)] \tag{18}$$

Expression (17) can be presented in the form

$$T_1 = \pi^{-1}P[-\theta - \sin\theta], \quad T_2 = \pi^{-1}P[-\theta + \sin\theta] \tag{19}$$

It is obvious from (19) that both principal stresses are always negative. Consequently, it is possible to construct a design with a reinforcement which is uniformly stressed and with identical principal strains

$$e_1 = e_2 = [illegible] = \text{const} \tag{20}$$

The trajectories of principal stresses, along which the reinforcement fibers are oriented, in this case, are families of cofocal ellipses and hyperbolae with foci at points

$$x = a, \quad y = 0, \qquad x = -a, \quad y = 0$$

here, the hyperbolae determine direction α.

On the x-axis segment [-a, a] the principal stresses are maximum, under condition (20) this indicates the maximality of value $S_1 + S_2$, then, according to (15), we find that

$$s_1 = s_2 T_1 \; 2P \; [illegible] \tag{21}$$

It is interesting to note that if the load on segment [-a, a] is not constant and the constant displacement v^l = const (smooth rigid stamp) with the same total force 2Pa, then the optimum design coincides with the constructed. Actually, the found deformation field permits us to satisfy condition

$$v(x, 0) = v^l \qquad (\varepsilon_x = \varepsilon x, \ \varepsilon_y = -\varepsilon, \ v = -\varepsilon y + v^l)$$

It is evident from (19), (21) that when $r_1, r_2 \to \infty$, $s_1, s_2 \to 0$, consequently, at the sufficient distance from the loaded segment the reinforcement can be terminated (when the contribution of depleted reinforcement to the total material strength proves to be small in comparison with the actual strength of the binder).

B. Circular plate loaded with a uniform tangential force along the internal and external contour. We construct a design which satisfies conditions (11). As a matter of course we introduce polar coordinates $r\theta$ and accept for the reinforcement directions condition

$$\gamma = \theta \pm 1/4\pi$$

(plus for α and minus for β)

specifying that direction α forms angle $1/4\pi$ with the radius-vector, and angle γ with axis x. In this case the equation of line α in system xy has the form

$$y' = (x + y) / (x - y)$$

A solution of the latter, written in polar coordinates $\theta = \ln cr$ (logarithmic spiral), is c = const. Equation of lines β: $\theta = -\ln cr$.

Families of such curves satisfy condition (14). The tensor components of stresses

$$T_{r\theta} = Fa^2 / r^2, \qquad T_r = T_\theta = 0$$

Here F – force per unit length of the internal contour, a – internal plate radius, the external radius can be arbitrary.

The principal stresses, thickness, and reinforcement levels are expressed by formulas

$$T_1 = T_2 = pa^2/r^2, \quad h = F/k\tau s_0, \quad s_1 = s_2 = s_0 a^2/r^2$$

From conditions (11) we can also determine the displacements along the radius and in the circumferential direction

$$u^r = 0, \quad u^\varphi = 2\varepsilon r \ln(r/a)$$

C. Circular plate under internal pressure. The radial and circumferential directions we take to be the reinforcement directions, then the equations of equilibrium and compatibility will assume the form

$$T_2 = (rT^1)', \quad \varepsilon_1 = (r\varepsilon_2)' \tag{22}$$

Here subscript 1 pertains to the radial conditions and subscript 2 – to the circumferential; the prime indicates differentiation with respect to r.

It is shown in [1] that in the absence of external pressure the construction of a design with $\varepsilon_1 = \varepsilon_2 = \text{const}$ is impossible according to (10). It is readily seen that construction of a design which satisfies conditions (11) is also impossible due to the fact that the second condition (22) is not satisfied.

We will assume that the reinforcement fibers are oriented, this leads to the following condition concerning the reinforcement level in radial direction [1]:

$$S_1 = \mu/r \quad (\mu = \text{const}) \tag{23}$$

Boundary conditions

$$T_1 = -P \text{ when } r = a, \quad T_1 = 0 \text{ when } r = b$$

Expressing all variables in terms of ε_2, we find specifically that

$$S \quad \mu(r\varepsilon_2'' + 2\varepsilon_2')/\varepsilon_2 \tag{24}$$

We select S_2 such that volume (9) of reinforcement is the smallest, i.e.,

$$\int_a^b \frac{r^2\varepsilon_2'' + 2r\varepsilon_2'}{\varepsilon_2}\,dr = \min \qquad (25)$$

The Euler variation equation has the form

$$r\varepsilon_2\varepsilon_2'' + 2\varepsilon_2\varepsilon_2' - r(\varepsilon_2')^2 = 0$$

Its general solution is $\varepsilon_2 = c_1 \exp(c_2/r)$; where $T_1 = 0$, $S_1 \neq 0$ when $r = b$; consequently (see (3), (5)), we have $\varepsilon_1(b) = 0$. From this $c_2 = b$. With aid of (22) we obtain

$$\varepsilon_2 = c_1 \exp(b/r) \qquad \varepsilon_1 = c_1(1 - b/r)\exp(b/r) \qquad (26)$$

Let $b \leq 2a$ (for $b > 2a$ the investigation is similar but the expression for constant c_1 is different).

Functions (26) have the greatest values when $r = a$, here

$$\max\{|\varepsilon_1|, |\varepsilon_2|\} = \varepsilon_2(a)$$

We will assume that this maximum is equal to the maximum deformation value ε and thus determine the constant

$$c_1 = \varepsilon \exp(-b/a)$$

Now, from (24) we determine the level of reinforcement $S_2(r)$, which yields the extremum to the examined functional

$$S_2(r) = \mu b^2/r^3$$

The remaining boundary condition we use to find constant μ finally,

$$S_1 = Pa^2/E\varepsilon(b-a)r, \qquad S_2 = Pa^2b^2/E\varepsilon(b-a)r^3$$
$$h = P(a^2+b^2)/E\varepsilon\sigma_*(b-a)a$$

The design is constructed. We can show that the second variation of functional (25) is an extremely positive functional

at "point" $\varepsilon_2 = c_1 \exp(b/r)$; consequently, the found extremum is a minimum.

For reinforcement volume (9) we obtain the following formula:

$$V^* = 2\pi Pq(a + b)/Ec$$

In conclusion, we will examine the behavior of representative point (ε_1, ε_2) describing the stressed state of the body elements along a certain plate radius. For this we will express ε_1 in terms of ε_2($b = 2a$) with the aid of (26)

$$\varepsilon_1 = -\varepsilon_2[1 + \ln(\varepsilon_2/c)]$$

A corresponding curve is shown in Fig. 2. When r changes from a to 2a the representing point travels the curve from B to Q.

As we can see, there is no analogy here with the optimum design of the rigidly elastic bodies, where it is most convenient and always possible to use the points on a restricting surface. In the examined case, the points at which the limiting curve intersects the axes of coordinates (for example) are less suitable than many internal points, since one of the reinforcement directions for them is not loaded.

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